NAG Toolbox for MATLAB

f01qg

1 Purpose

f01qg reduces the m by n ($m \leq n$) real upper trapezoidal matrix A to upper triangular form by means of orthogonal transformations.

2 Syntax

```
[a, zeta, ifail] = f01qg(a, 'm', m, 'n', n)
```

3 Description

The m by n ($m \leq n$) real upper trapezoidal matrix A given by

$$A = \begin{pmatrix} U & X \end{pmatrix},$$

where U is an m by m upper triangular matrix, is factorized as

$$A = \begin{pmatrix} R & 0 \end{pmatrix} P^{\mathrm{T}},$$

where P is an n by n orthogonal matrix and R is an m by m upper triangular matrix.

P is given as a sequence of Householder transformation matrices

$$P = P_m \cdots P_2 P_1,$$

the $(m-k+1)$th transformation matrix, P_k, being used to introduce zeros into the kth row of A. P_k has the form

$$P_k = \begin{pmatrix} I & 0 \\ 0 & T_k \end{pmatrix},$$

where

$$T_k = I - u_k u_k^{\mathrm{T}},$$

$$u_k = \begin{pmatrix} \zeta_k \\ 0 \\ z_k \end{pmatrix},$$

ζ_k is a scalar and z_k is an $(n-m)$ element vector. ζ_k and z_k are chosen to annihilate the elements of the kth row of X.

The vector u_k is returned in the kth element of the array **zeta** and in the kth row of **a**, such that ζ_k is in **zeta**(k) and the elements of z_k are in $\mathbf{a}(k, m+1), \ldots, \mathbf{a}(k, n)$. The elements of R are returned in the upper triangular part of **a**.

For further information on this factorization and its use see Section 6.5 of Golub and Van Loan 1996.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Wilkinson J H 1965 *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

5 Parameters

5.1 Compulsory Input Parameters

1: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The leading m by n upper trapezoidal part of the array **a** must contain the matrix to be factorized.

5.2 Optional Input Parameters

1: **m – int32 scalar**

m, the number of rows of the matrix A.

When $\mathbf{m} = 0$ then an immediate return is effected.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq \mathbf{m}$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda

5.4 Output Parameters

1: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by m upper triangular part of **a** will contain the upper triangular matrix R, and the m by $(n - m)$ upper trapezoidal part of **a** will contain details of the factorization as described in Section 3.

2: **zeta(∗) – double array**

Note: the dimension of the array **zeta** must be at least $\max(1, \mathbf{m})$.

zeta(k) contains the scalar ζ_k for the $(m - k + 1)$th transformation. If $T_k = I$ then **zeta**$(k) = 0.0$, otherwise **zeta**(k) contains ζ_k as described in Section 3 and ζ_k is always in the range $\left(1.0, \sqrt{2.0}\right)$.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= -1$

On entry, $\mathbf{m} < 0$,
or $\mathbf{n} < \mathbf{m}$,
or **lda** $< \mathbf{m}$.

7 Accuracy

The computed factors R and P satisfy the relation

$$(R0)P^{\mathrm{T}} = A + E,$$

where

$$\|E\| \leq c\epsilon\|A\|,$$

ϵ is the ***machine precision*** (see x02aj), c is a modest function of m and n and $\|.\|$ denotes the spectral (two) norm.

8 Further Comments

The approximate number of floating-point operations is given by $2m^2(n-m)$.

9 Example

```
a = [2.4, 0.8, -1.4, 3, -0.8;
     0, 1.6, 0.8, 0.4, -0.8;
     0, 0, 1, 2, 2];
[aOut, zeta, ifail] = f01qg(a)
```

```
aOut =
   -4.0000   -1.0000   -1.0000    0.6325   -0.0000
         0   -2.0000    0.0000    0.0000   -0.4472
         0         0   -3.0000    0.5774    0.5774
zeta =
    1.2649
    1.3416
    1.1547
ifail =
           0
```